represent targets that have acquired 0.1% of the residual kinetic energy of the projectile and have masses 100 and 1000 times as large as that of the projectile. These systems have a ratio of momentum to kinetic energy of 316:1 and 1000:1, respectively. In systems with large differences in the relative mass of the components, one should be cautious before assuming that the momentum and kinetic energy of any one component is negligible.

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Comment on "Angle of Attack from Body-Fixed Rate Gyros"

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THE condition $\dot{\alpha} = 0$ when $\dot{\Omega} = 0$, which Nelson[1] used to develop a technique for measuring the angle of attack of a symmetric, spinning body, can be alternately derived without algebraic manipulation. A clearer physical interpretation of the vehicle motion is obtained by the alternate method.

Instead of the Euler equations, use the equation of motion derived by Nidey and Seames[2] for a coordinate system rotating with the transverse angular velocity:

$$M_\tau = I_\tau \dot{\omega}_\tau \tag{1}$$

$$M_\nu = I_\tau \omega_\tau \Omega_\lambda - I_\lambda \omega_\lambda \omega_\tau \tag{2}$$

$$M_\lambda = I_\lambda \dot{\omega}_\lambda \tag{3}$$

where $\dot{\nu}_\lambda \equiv \dot{\Omega}$. We obtain the result $\dot{\Omega} = 0$ when $\epsilon = \phi$ (or $\epsilon = \phi + 180°$) by inspection of Eq. (1) since the latter equality implies $M_\tau = 0$. Thus, the need for algebraic manipulation is eliminated.

A symmetric body at a preatmospheric altitude precesses with the well-known free-body motion. After entering the atmosphere the body is constrained by the influence of the aerodynamic moment to precess about an axis other than that formed by the moment of momentum vector. In fact, the body precesses with the angular velocity $\boldsymbol{\omega}_p$ (or $\boldsymbol{\Omega}$ in notation of Ref. 2) the same as the τ, ν, λ coordinate system. Because the coordinates and the body share the transverse component of angular velocity

$$\boldsymbol{\omega}_p = \Omega_\lambda \boldsymbol{\lambda} + \omega_\tau \boldsymbol{\tau} \tag{4}$$

where Ω_λ can be found from Eq. (2).

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Comment on "Motion of the Center of Gravity of a Variable-Mass Body"

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IN a recent note,[1] Punga has shown the effects of a moving center of mass on the equation of motion of a variable mass body. The result that Punga was seeking can be found in the literature,[2–5] but his result disagrees with previously published results. It is this author's contention that the premise upon which Punga's paper was based is false.

Punga's basic premise was that the equation of motion for a variable-mass system can be expressed in the following form:

$$\int (d^2\mathbf{R}/dt^2) dm = \mathbf{F} + \mathbf{K} \tag{1}$$

where the integration is extended over the mass of the body at time t. He defines $\mathbf{F}$ as the external force acting on the body and $\mathbf{K}$, the reactive force acting on the body which is produced by mass ejection. Equation (1) is an extension of Newton's second law to any body, but should not include the term $\mathbf{K}$ since this imaginary force is a by-product of the left-hand side of the equation.

Thorpe[2] has derived a relation for the motion of the center of mass of a variable-mass body in which he started with the classical formulation of Newton's second law, namely,

$$\mathbf{F} = \int \mathbf{a}\, dm \tag{2}$$

where $\mathbf{a}$ is the acceleration of the element of mass dm. Thorpe's result is as follows:

$$\mathbf{F} = \frac{d}{dt}(M\mathbf{V}^*) + \int_s \rho \mathbf{u}(\mathbf{u} - \mathbf{v})\cdot\mathbf{n}\, ds + \frac{d}{dt}\int_s \rho(\mathbf{r} - \mathbf{R}^*)(\mathbf{u} - \mathbf{v})\cdot\mathbf{n}\, ds \tag{3}$$

where $\mathbf{V}^*$ is the velocity of the mass center, $\mathbf{u}$ is the absolute velocity of the mass particle, and $\mathbf{v}$ is the velocity of the boundary. Let us denote the absolute velocity of the escaping gases as $\mathbf{v}_e$, and from physical reasoning the mass rate of flow across the boundary can be written as

$$\frac{dM}{dt} = -\int_s \rho(\mathbf{u} - \mathbf{v})\cdot\mathbf{n}\, ds \tag{4}$$

Let us define a new quantity

$$\mathbf{R}_N \frac{dM}{dt} = \int_s \rho \mathbf{r}(\mathbf{u} - \mathbf{v})\cdot\mathbf{n}\, ds \tag{5}$$

Then with these new definitions, Eq. (3) can be reduced to

$$\mathbf{F} = M\frac{d\mathbf{V}^*}{dt} + \frac{d^2M}{dt^2}(\mathbf{R}^* - \mathbf{R}_N) + \frac{dM}{dt}\left(2\frac{d\mathbf{R}^*}{dt} - \frac{d\mathbf{R}_N}{dt} - \mathbf{v}_e\right) \tag{6}$$

Equation (6) has been derived by Rankin[3] and Leitmann[4] by using an alternate form of Newton's second law.

Equation (6) can be reduced further by using an intermediate frame of reference fixed in the body at point 0, in a manner

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similar to that used by Punga. Using the Nomenclature of Punga, the following velocities can be written:

$$\left.\begin{aligned} \frac{d\mathbf{R}^{(G)}}{dt} &= \frac{d\mathbf{R}^{(0)}}{dt} + \omega \times \mathbf{r}^{(G)} + \frac{\delta \mathbf{r}^{(G)}}{\delta t} \\ \frac{d\mathbf{R}_N}{dt} &= \frac{d\mathbf{R}^{(0)}}{dt} + \omega \times \mathbf{r}_N + \frac{\delta \mathbf{r}_N}{\delta t} \\ \mathbf{v}_e &= \frac{d\mathbf{R}^{(0)}}{dt} + \omega \times \mathbf{r}_N + \mathbf{v}_e^{(0)} \end{aligned}\right\} \quad (7)$$

where $\mathbf{v}_e^{(0)}$ is the velocity of the escaping gases relative to the reference systems fixed in the body. Using these relations, Eq. (6) will reduce to

$$\bar{F} = M\frac{d\mathbf{V}^*}{dt} - \frac{d^2M}{dt^2}(\mathbf{r}_N - \mathbf{r}^{(G)}) - \frac{dM}{dt}\left\{\mathbf{v}_e^{(0)} + \frac{\delta \mathbf{r}_N}{\delta t} - 2\frac{\delta \mathbf{r}^{(G)}}{\delta t} - 2\,\omega \times (\mathbf{r}_N - \mathbf{r}^{(G)})\right\} \quad (8)$$

The last two terms of Eq. (8) are usually labeled as the reactive force since they are functions of mass ejection. Now if we call the reactive force **K**, then Eq. (8) reduces to

$$\mathbf{F} + \mathbf{K} = M(d\mathbf{V}^*/dt) \quad (9)$$

which is the usual form of the rocket equation. Examining Eq. (8), the motion of the center of mass cannot be separated from the reactive force.

There are a few printing errors that crept into Punga's paper, which do not affect the final equation derived.

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Spectroscopic Constants for the N^+ Ion

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THE purpose of this note is to point out an error in Ref. 1. Stupochenko, et al., in their Table 1, list incorrect spectroscopic data for the ion N^+. The authors cite Ref. 2 as the source of their data, and their error stems from the fact that they transcribed Moore's data for N^{++} (NIII) rather than that for N^+ (NII).

The data in Table 1, from Ref. 2, should be substituted for that in Ref. 1. As an example of the magnitude of error involved in using Ref. 1 rather than Ref. 2, we compute the partition function of N^+ at several temperatures (see Table 2).

$$Q_{N+} = \sum_i g_i\, e^{-\epsilon_i/kT}$$

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Table 1 Energy levels for N^+ ion

ϵ_p(cm^{-1})	g_p
0	1
49.1	3
131.3	5
15315.7	5
32687.1	1
47167.7	5
92237.9	7
92251.3	5
92252.9	3

Table 2 N^+ partition function

	Q_{N^+}	
T(°K)	Ref. 1	Ref. 2
1,000	5.11	7.94
6,000	5.84	8.93
12,000	5.92	9.72

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Viscous Flow Properties on Slender Cones

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Nomenclature

M = Mach number
P = static pressure
Re = Reynolds number
T = temperature
U = velocity
β = $(M_1^2 - 1)^{0.5}$
δ^* = boundary-layer displacement thickness
γ = ratio of specific heats
θ = semivertex angle
σ = Prandtl number

Subscripts

1 = freestream conditions
2 = condition at outer edge of boundary layer
c = inviscid conical flow value
w = wall
x = distance from vertex along cone surface

THE use of a similarity parameter has been employed by several authors to predict the inviscid flow properties for supersonic and hypersonic flow over cones. Linnell and

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